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Generalized Quadrature Formulas of Birkhoff-Young Type

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Abstract: The well-known quadrature formula for numerical integration of analytic and harmonic functions in the complex domain over the line segment $[a-h, a+h]$,

$$\int_{a-h}^{a+h} f(z)\mathrm{d}z = \frac{h}{15}\{24f(a) + 4[f(a+h) + f(a-h)] - [f(a+\mathrm{i}h) + f(a-\mathrm{i}h)]\} + R(f),$$

was obtained in 1950 by Birkhoff and Young [1], with the error estimate

$$|R(f)| \leq \frac{|h|^7}{1890} \max_{z\in S} \left|f^{(6)}(z)\right|,$$

where S denotes the square with vertices $a + \mathrm{i}^k h, k = 0,1,2,3$. In our lecture we consider several types of quadratures of Birkhoff-Young type developed to date (see [2]–[14]), as well as a sequence of the weighted generalized quadrature rules of the maximal degree of exactness and their connection with multiple orthogonal polynomials. A characterization and uniqueness of such rules are presented, as well as the numerical construction of nodes and weight coefficients.

Keywords: Quadrature formula, Weight function, Error estimate, ortogonality, multiple orthogonal polynomials.

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